1 The planet Venus may be considered to be a sphere of uniform density $5.24 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$. The gravitational field strength at the surface of Venus is $8.87 \mathrm{~N} \mathrm{~kg}^{-1}$.
(a) (i) Show that the gravitational field strength $g_{\mathrm{s}}$ at the surface of a planet is related to the the density $\rho$ and the radius $R$ of the planet by the expression

$$
g_{s}=\frac{4}{3} \pi G R \rho
$$

where $G$ is the gravitational constant.
(ii) Calculate the radius of Venus.

Give your answer to an appropriate number of significant figures.
$\qquad$
radius $=$ m
(b) At a certain time, the positions of Earth and Venus are aligned so that the distance between them is a minimum.
Sketch a graph on the axes below to show how the magnitude of the gravitational field strength $g$ varies with distance along the shortest straight line between their surfaces. Consider only the contributions to the field produced by Earth and Venus.
Mark values on the vertical axis of your graph.


2 (a) State, in words, Newton's law of gravitation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) By considering the centripetal force which acts on a planet in a circular orbit, show that $T^{2} \propto R^{3}$, where $T$ is the time taken for one orbit around the Sun and $R$ is the radius of the orbit.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) The Earth's orbit is of mean radius $1.50 \times 10{ }^{11} \mathrm{~m}$ and the Earth's year is 365 days long.
(i) The mean radius of the orbit of Mercury is $5.79 \times 10^{10} \mathrm{~m}$. Calculate the length of Mercury's year.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) Neptune orbits the Sun once every 165 Earth years.

Calculate the ratio $\frac{\text { distance from Sun to Neptune }}{\text { distance from Sun to Earth }}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

3 (a) Give two examples of the techniques used by geologists to obtain values of the strength of the local gravitational field of the Earth.
In each of your quoted examples, describe the information that the geologists can derive from their measurements.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

In 1774, Nevil Maskelyne carried out an experiment near the mountain of Schiehallion in Scotland to determine the density of the Earth.

Figure 1 shows two positions of a pendulum hung near to, but on opposite sides of, the mountain. The centre of mass of the mountain is at the same height as the pendulum.

Figure 1

(b) (i) Explain why the pendulums do not point towards the centre of the Earth.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) Suggest why Maskelyne carried out the experiment on both sides of the mountain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Figure 2 shows measurements made with the left-hand pendulum in Figure 1.

## Figure 2


(i) The mountain is in the appropriate shape of a cone 0.50 km high and 1.3 km base radius; it rises from a locally flat plain.
Show that the mass of the mountain is about $2 \times 10^{12} \mathrm{~kg}$.
volume of a cone $=\frac{1}{3} \pi r^{2} h$
density of rock $=2.5 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$
(ii) Figure 2 shows the left-hand pendulum bob lying on a horizontal line that also passes through the centre of mass of the mountain. The bob is 1.4 km from the centre of the mountain and it hangs at an angle of $0.0011^{\circ}$ to the vertical.

Calculate the mass of the Earth.
(iii) The answer Maskelyne obtained for the mass of the Earth was lower than today's accepted value even though he had an accurate value for the Earth's radius.

Suggest one reason why this should be so.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(Total 14 marks) surface it is effectively uniform, as shown in Figure 2.

Alongside each figure, sketch a graph to show how the gravitational potential $V$ associated with the planet varies with distance $r$ (measured outwards from the surface of the planet) in each of these cases.


Figure 1


Figure 2
(Total 4 marks)
The Rosetta space mission placed a robotic probe on Comet 67P in 2014.
(a) The total mass of the Rosetta spacecraft was 3050 kg . This included the robotic probe of mass 108 kg and 1720 kg of propellant. The propellant was used for changing velocity while travelling in deep space where the gravitational field strength is negligible.

Calculate the change in gravitational potential energy of the Rosetta spacecraft from launch until it was in deep space.
Give your answer to an appropriate number of significant figures.
Mass of the Earth $=6.0 \times 10^{24} \mathrm{~kg}$
Radius of the Earth $=6400 \mathrm{~km}$
$\qquad$ J
(b) As it approached the comet, the speed of the Rosetta spacecraft was reduced to match that of the comet. This was done in stages using four 'thrusters'. These were fired simultaneously in the same direction.

Explain how the propellant produces the thrust.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Each thruster provided a constant thrust of 11 N .

Calculate the deceleration of the Rosetta spacecraft produced by the four thrusters when its mass was 1400 kg .
decleration $\qquad$ $\mathrm{m} \mathrm{s}^{-2}$
(d) Calculate the maximum change in speed that could be produced using the 1720 kg of propellants.

Assume that the speed of the exhaust gases produced by the propellant was $1200 \mathrm{~m} \mathrm{~s}^{-1}$
$\qquad$ $\mathrm{m} \mathrm{s}^{-1}$
(e) When the robotic probe landed, it had to be anchored to the comet due to the low gravitational force. Comet 67 P has a mass of about $1.1 \times 10^{13} \mathrm{~kg}$. A possible landing site was about 2.0 km from the centre of mass.
(i) Calculate the gravitational force acting on the robotic probe when at a distance of 2.0 km from the centre of mass of the comet.
gravitational force $\qquad$ N
(ii) Calculate the escape velocity for an object 2.0 km from the centre of mass of the comet.
escape velocity $\qquad$ $\mathrm{m} \mathrm{s}^{-1}$
(iii) A scientist suggests using a drill to make a vertical hole in a rock on the surface of the comet. The anchoring would be removed from the robotic probe before the drill was used. The drill would exert a force of 25 N for 4.8 s .

Explain, with the aid of a calculation, whether this process would cause the robotic probe to escape from the comet.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Satellites used for telecommunications are usually in geostationary orbits. Using suitable dishes to transmit the signals, communication over most of the Earth's surface is possible at all times using only 3 satellites.

Satellites used for meteorological observations and observations of the Earth's surface are usually in low Earth orbits. Polar orbits, in which the satellite passes over the North and South Poles of the Earth, are often used.

One such satellite orbits at a height of about 12000 km above the Earth's surface circling the Earth at an angular speed of $2.5 \times 10^{-4} \mathrm{rad} \mathrm{s}^{-1}$. The microwave signals from the satellite are transmitted using a dish and can only be received within a limited area, as shown in the image below.

not to scale

The signal of wavelength $\lambda$ is transmitted in a cone of angular width $\theta$, in radian, given by

$$
\theta=\frac{\lambda}{d}
$$

where $d$ is the diameter of the dish.
The satellite transmits a signal at a frequency of 1100 MHz using a 1.7 m diameter dish. As this satellite orbits the Earth, the area over which a signal can be received moves. There is a maximum time for which a signal can be picked up by a receiving station on Earth.
(a) Describe two essential features of the orbit needed for the satellite to appear geostationary.
$\qquad$
$\qquad$
(b) Calculate the time taken, in s, for the satellite mentioned in line 7 in the passage to complete one orbit around the Earth.
$\qquad$
(c) Show that at a distance of 12000 km from the satellite the beam has a width of 1900 km .
(d) The satellite is in a polar orbit and passes directly over a stationary receiver at the South Pole.

Show that the receiver can remain in contact with the satellite for no more than about 20 minutes each orbit. radius of the Earth $=6400 \mathrm{~km}$
maximum time $=$ $\qquad$ minute
(e) The same satellite is moved into a higher orbit.

Discuss, with reasons, how this affects the signal strength and contact time for the receiver at the South Pole.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

7 (a) (i) State what is meant by the term escape velocity.
(ii) Show that the escape velocity, $v$, at the Earth's surface is given by $v=\sqrt{\frac{2 G M}{R}}$ where $M$ is the mass of the Earth and $R$ is the radius of the Earth.
(iii) The escape velocity at the Moon's surface is $2.37 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}$ and the radius of the Moon is $1.74 \times 10^{6} \mathrm{~m}$.

Determine the mean density of the Moon.
mean density $\qquad$ $\mathrm{kg} \mathrm{m}^{-3}$
(b) State two reasons why rockets launched from the Earth's surface do not need to achieve escape velocity to reach their orbit.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

8 The Global Positioning System (GPS) is a system of satellites that transmit radio signals which can be used to locate the position of a receiver anywhere on Earth.

(a) A receiver at sea level detects a signal from a satellite in a circular orbit when it is passing directly overhead as shown in the diagram above.
(i) The microwave signal is received 68 ms after it was transmitted from the satellite. Calculate the height of the satellite.
$\qquad$
$\qquad$
(ii) Show that the gravitational field strength of the Earth at the position of the satellite is $0.56 \mathrm{~N} \mathrm{~kg}^{-1}$.
mass of the Earth $\quad=6.0 \times 10^{24} \mathrm{~kg}$
mean radius of the Earth $=6400 \mathrm{~km}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) For the satellite in this orbit, calculate
(i) its speed,
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) its time period.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

9 A planet has a radius half the Earth's radius and a mass a quarter of the Earth's mass. What is the approximate gravitational field strength on the surface of the planet?

A $\quad 1.6 \mathrm{~N} \mathrm{~kg}^{-1} \quad \square$

B $\quad 5.0 \mathrm{~N} \mathrm{~kg}^{-1} \quad \circ$

C $\quad 10 \mathrm{~N} \mathrm{~kg}^{-1} \quad \square$

D $\quad 20 \mathrm{~N} \mathrm{~kg}^{-1} \quad \square$
(Total 1 mark)

10 Two stars of mass $M$ and $4 M$ are at a distance $d$ between their centres.


The resultant gravitational field strength is zero along the line between their centres at a distance $y$ from the centre of the star of mass $M$.

What is the value of the ratio $\frac{y}{d}$ ?

A $\frac{1}{2}$


B $\frac{1}{3}$


C $\frac{2}{3}$


D $\frac{3}{4}$

(Total 1 mark)
11
Which of the following statements about Newton's law of gravitation is correct? Newton's gravitational law explains

A the origin of gravitational forces.

B why a falling satellite burns up when it enters the Earth's atmosphere.


C why projectiles maintain a uniform horizontal speed.

D how various factors affect the gravitational force between two particles.


12 Two planets $\mathbf{X}$ and $\mathbf{Y}$ are in concentric circular orbits about a star $\mathbf{S}$. The radius of the orbit of $\mathbf{X}$ is $R$ and the radius of orbit of $\mathbf{Y}$ is $2 R$.


The gravitational force between $\mathbf{X}$ and $\mathbf{Y}$ is $F$ when angle $\mathbf{S X Y}$ is $90^{\circ}$, as shown in the diagram. What is the gravitational force between $\mathbf{X}$ and $\mathbf{Y}$ when they are nearest to each other?

A $\quad 2 F$
B $\quad 3 F$
C $\quad 4 F$
D $\quad 5 F$
(Total 1 mark)
13 X and Y are two stars of equal mass $M$. The distance between their centres is $d$.


What is the gravitational potential at the mid-point $P$ between them?

A $\frac{G M}{2 d}$
B $-\frac{G M}{d}$
C $-\frac{4 G M}{d}$
D $-\frac{8 G M}{d}$

A geosynchronous satellite is in a constant radius orbit around the Earth. The Earth has a mass of $6.0 \times 10^{24} \mathrm{~kg}$ and a radius of $6.4 \times 10^{6} \mathrm{~m}$.

What is the height of the satellite above the Earth's surface?
A $\quad 1.3 \times 10^{7} \mathrm{~m}$
B $\quad 3.6 \times 10^{7} \mathrm{~m}$
C $\quad 4.2 \times 10^{7} \mathrm{~m}$
D $\quad 4.8 \times 10^{7} \mathrm{~m}$
(a) (i) $\quad M=\frac{4}{3} \pi R^{3} \rho \checkmark$
combined with $g_{\mathrm{s}}=\frac{G M}{R^{2}}$ (gives $\left.g_{\mathrm{s}}=\frac{4}{3} \pi G R \rho\right) \checkmark$
Do not allow rinstead of $R$ in final answer but condone in early stages of working.
Evidence of combination, eg cancelling $R^{2}$ required for second mark.
(ii) $\quad R=\left(\frac{3 g_{s}}{4 \pi G \rho}\right)=\frac{3 \times 8.87}{4 \pi 6.67 \times 10^{-11} \times 5.24 \times 10^{3}} \checkmark$
gives $R=6.06 \times 10^{6}(\mathrm{~m}) \checkmark$
answer to 3SF $\checkmark$
SF mark is independent but may only be awarded after some working is presented.

3
(b) line starts at 9.81 and ends at $8.87 \checkmark$
correct shape curve which falls and rises $\checkmark$
falls to zeo value near centre of and to right of centre of distance scale $\checkmark$
[Minimum of graph in 3rd point to be $>0.5$ and $<0.75$ SE-SV distance]


For 3rd mark accept flatter curve than the above in central region.

2 (a) attractive force between point masses (1)
proportional to (product of) the masses (1)
inversely proportional to square of separation/distance apart (1)
(b) $m \omega^{2} R=(-) \frac{G M m}{R^{2}}\left(\right.$ or $\left.=\frac{m v^{2}}{R}\right)$ (1
(use of $T=\frac{2 \pi}{\omega}$ gives) $\frac{4 \pi^{2}}{T^{2}}=\frac{G M}{R^{3}}$
$G$ and $M$ are constants, hence $T^{2} \propto R^{3}(1)$
(c) (i) (use of $T^{2} \propto R^{3}$ gives) $\frac{365^{2}}{\left(1.50 \times 1 \square^{11}\right)^{3}}=\frac{T_{m}^{2}}{\left(5.79 \times 10^{10}\right)^{3}}$

$$
T_{\mathrm{m}}=87(.5) \text { days (1) }
$$

(ii) $\frac{1^{2}}{\left(1.50 \times 10^{11}\right)^{3}}=\frac{165^{2}}{R_{\mathrm{N}}^{3}}$ (1) (gives $\left.R_{\mathrm{N}}=4.52 \times 10^{12} \mathrm{~m}\right)$

$$
\text { ratio }=\frac{4.51 \times 10^{12}}{1.50 \times 10^{11}}=30(.1)
$$

3 (a) technique one (1)
information derived from it (1)
technique two (1)
information derived from it (1)
(b) (i) gravitational attraction to ...(1)
...centre of gravity(mass) of mountain (1)
(ii) cancellation of some systematic errors (1)
(c) (i) calculates volume of cone (1)
mass $=$ density $\times$ volume seen (1)
$2.2 \times 10^{12} \mathrm{~kg}(1)$
(ii) $\quad$ sideways force $/ \mathrm{mg}=\tan (0.0011)(1)$
sideways force $=\mathrm{Gm}_{\text {sch }} 0.5 /(1400)^{2}$ subst seen (1)
$2.4 \times 10^{24} \mathrm{~kg}(1)$
(iii) his density estimate was too low (1) or mean density of the Earth is higher than that of the mountain (1)

4

gradient decreases as $r$ increases (1)
$V$ increases as $r$ increases (1)
only negative values of $V$ shown (1)

constant gradient (1)
$V$ increases as $r$ increases (1)

5 (a) Total mass of spacecraft $=3050 \mathrm{~kg}$
Change in $\mathrm{PE}=\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 3050}{6400 \times 10^{3}}$
$1.9 \times 10^{11}(\mathrm{~J})$
2 sf
condone errors in powers of 10 and incorrect mass for payload
Allow if some sensible working
(b) Chemical combustion of propellant / fuel or gases produced at high pressure

Gas is expelled / expands through nozzle
Change in momentum of gases escaping
equal and opposite change in momentum of the spacecraft
Thrust = rate of change of change in momentum
Max 3
N3 in terms of forces worth 1

3
(c) $\quad 0.031(4)\left(\mathrm{m} \mathrm{s}^{-2}\right)$
(d) Use of rocket equation
$v=1200 \ln \frac{3050}{1330}$
$996\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$
Condone $1000\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$
(e) (i) Use of correct mass 108 kg
$F=\frac{6.67 \times 10^{-11} \times 1.1 \times 10^{13} \times 108}{\left(2 \times 10^{3}\right)^{2}}$
0.0198 N

Allow incorrect powers of 10 and mass
3
(ii) Use of $v=\sqrt{\frac{2 G M}{r}}$

Correct substitution $v=\frac{2 \times 6.67 \times 10^{-11} \times 1.1 \times 10^{18}}{2 \times 10^{8}}$
$0.86\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$
Recognisable mass - condone incorrect power of 10
3
(iii) Impulse $=25 \mathrm{~N} \times 4.8=120 \mathrm{~N} \mathrm{~s}$
$(120=108 v$ so $)$ Velocity $=1.1 \mathrm{~m} \mathrm{~s}^{-1}$
Clear conclusion
ie explanation/comparison of calculated velocity with escape velocity from (e)(ii)

May use $F=$ ma approach

6 (a) Equatorial orbit $\checkmark$
Moving west to east $\checkmark$
Period 24 hours $\sqrt{ }$

## ANY TWO

(b) $\quad T\left(=\frac{2 \pi}{\omega}=\frac{2 \pi}{2.5(4) \times 10^{-4}}\right)=2.5 \times 10^{4} \mathrm{~s} \checkmark$
(c) $\left.\quad \lambda\left(=\frac{c}{f}=\frac{3.0 \times 10^{8}}{1100 \times 10^{6}}\right)=0.27(3) \mathrm{m}\right) \checkmark$
$\theta\left(=\frac{\lambda}{d}=\frac{0.27(3)}{1.7}\right)=0.16(1) \mathrm{rad}=92^{\circ} \checkmark$
(linear) width $=D \theta=12000 \mathrm{~km} 0.16(1) \mathrm{rad})=1.9(3) \times 10^{3} \mathrm{~km} \checkmark$
(d) Angle subtended by beam at Earth's centre
$=$ beam width $/$ Earth's radius $\left.=1.9(3) \times 10^{3} / 6400\right) \checkmark$
$0.30 \mathrm{rad}\left(\right.$ or $\left.17^{\circ}\right) \checkmark$
Time taken $=\alpha / \omega=0.30 / 2.5(4) \times 10^{-4}=1.18 \times 10^{3} \mathrm{~s}$
$=20 \mathrm{mins} \checkmark$
Alternative:
Speed of point on surface directly below satellite $=\omega R$

$$
\begin{aligned}
& \left.=2.5(4) \times 10^{-4} \times 6400 \times 10^{3}\right) \\
& =1.63 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1} \sqrt{ }
\end{aligned}
$$

Time taken $=$ width $/$ speed

$$
\begin{aligned}
& =1.93 \times 10^{6} \mathrm{~m} / 1.63 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1} \\
& =1.18 \times 10^{3} \mathrm{~s}
\end{aligned}
$$

(accept $1.2 \times 10^{3}$ s or 20 mins ) $\checkmark$
or
Satellite has to move through angle of $1900 / 6400$ radian $=0.29$ rad $\sqrt{ }$
Fraction of one orbit $=0.30 / 2 \times 3.14 \sqrt{ }$
Time $=0.048 \times 2.5 \times 10^{4}=1.19 \times 10^{3} \mathrm{~s} \checkmark$
Time $=\frac{17}{360} \times 2.5 \times 10^{4}=1.18 \times 10^{3} \mathrm{~s}$
or
Circumference of Earth $=2 \pi \times 6370 \checkmark$

$$
=40023 \mathrm{~km}
$$

Width of beam at surface $=1920 \mathrm{~km} \checkmark$
Time $=\frac{1920}{40023} \times 2.48 \times 10^{4}$

$$
=1180 \mathrm{~s}=19.6 \mathrm{~min} \checkmark
$$

(e) Signal would be weaker $\checkmark$ (as distance it travels is greater)

Energy spread over wider area/intensity decreases with increase of distance $\checkmark$
Signal received for longer (each orbit) $\checkmark$
Beam width increases with satellite height/satellite moves at lower angular speed $\checkmark$ )
(a) (i) (Minimum) Speed (given at the Earth's surface) that will allow an object to leave / escape the (Earth's) gravitational field (with no further energy input)

Not gravity
Condone gravitational pull / attraction
(ii) $1 / 2 m v^{2}=\frac{G M m}{r}$

B1
Evidence of correct manipulation
At least one other step before answer
B1
(iii) Substitutes data and obtains $M=7.33 \times 10^{22}(\mathrm{~kg})$
or
Volume $=\left(1.33 \times 3.14 \times\left(1.74 \times 10^{6}\right)^{3}\right.$ or $2.2 \times 10^{19}$

$$
\text { or } \rho=\frac{3 v^{2}}{8 \pi G r^{2}}
$$

$3300\left(\mathrm{~kg} \mathrm{~m}^{-3}\right)$
(b) (Not given all their KE at Earth's surface) energy continually added in flight / continuous thrust provided / can use fuel (continuously)

Less energy needed to achieve orbit than to escape from Earth's gravitational field / it is not leaving the gravitational field

8 (a) (i) $h(=c t)\left(=3.0 \times 10^{8} \times 68 \times 10^{-3}\right)=2.0(4) \times 10^{7} \mathrm{~m}(1)$
(ii) $g=(-) \frac{G M}{r^{2}}$ (1)
$r\left(=6.4 \times 10^{6}+2.04 \times 10^{7}\right)=2.68 \times 10^{7}(\mathrm{~m})(1)$
(allow C.E. for value of $h$ from (i) for first two marks, but not 3rd)
$g=\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{\left(2.68 \times 10^{7}\right)^{2}}$ (1) $\quad\left(=0.56 \mathrm{~N} \mathrm{~kg}^{-1}\right)$
(b) (i) $g=\frac{v^{2}}{r}$
$v=\left[0.56 \times\left(2.68 \times 10^{7}\right)\right]^{1 / 2}(1)$
$=3.9 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}(1) \quad\left(3.87 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}\right)$
(allow C.E. for value of $r$ from a(ii)

$v=\left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{2.68 \times 10^{7}}\right)^{1 / 2}$
$\left.=3.9 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}(1)\right]$
(ii) $T\left(=\frac{2 \pi}{v}\right)=\frac{2 \pi \times 2.68 \times 10^{7}}{3.87 \times 10^{3}}$
$=4.3(5) \times 10^{4} \mathrm{~s}(1) \quad$ (12.(1) hours)
(use of $v=3.9 \times 10^{3}$ gives $T=4.3(1) \times 10^{4} \mathrm{~s}=12.0$ hours)
(allow C.E. for value of $v$ from (I)
[alternative for (b):
(i) $\nu\left(\frac{2 \pi}{T}\right)=\frac{2 \pi \times 2.68 \times 10^{7}}{4.36 \times 10^{4}}$
$\left.=3.8(6) \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}(1)\right]$
(allow C.E. for value of $r$ from (a)(ii) and value of $T$ )
(ii) $T^{2}=\left(\frac{4 \pi^{2}}{G M}\right) r^{3}$

$$
\begin{aligned}
& \left(=\frac{4 \pi^{2}}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}} \times\left(2.68 \times 10^{7}\right)^{3}\right)=\left(1.90 \times 10^{9}\left(\mathrm{~s}^{2}\right)(1)\right. \\
& T=4.3(6) \times 10^{4} \mathrm{~s}(\mathbf{1})
\end{aligned}
$$

## $9^{C}$

## Examiner reports

1
Combining $g=G M / R^{2}$ with $M=4 / 3 \pi R^{3} \rho$ caused very few problems in part (a)(i) and the marks were high. The main failing was the indiscriminate use of $r$ for $R$ in the working. In part (a)(ii) the correct substitution of the given values into the equation from (a)(i) readily produced the expected answer, which was usually given correctly to three significant figures. A minority of the students chose to ignore the equation from part (a)(i) and made hard work for themselves by working out a result from first principles.

In part (b) the graph of $g$ against the distance between Earth and Venus was rewarding for most. A graph of a correct general shape was usually presented, with the majority appreciating that there would be a minimum value to the right of centre. This minimum was not always shown to be zero, which was expected. Some answers did not heed the instruction to mark values on the vertical axis of the graph.

2 It was rare for all three marks to be awarded in part (a). Most answers made at least some reference to the proportionality and inverse proportionality involved in Newton's law, but references to point masses or to the attractive nature of the force were scarce.
The essential starting point in part (b) was a correct statement equating the gravitational force with $m \omega^{2} R$; the more able candidates had little difficulty in then applying $T=2 \pi / \omega$ to derive the required result, and three marks were usually obtained by them.

Both halves of part (c) followed directly from the $T^{2} \propto R^{3}$ result in part (b), and the candidates who realised this usually made excellent progress. Unfortunately, a large proportion tried to go back to first principles and tied themselves in knots with the algebra and/or arithmetic, often getting nowhere. Confusion over which unit of time to employ in the different parts caused much difficulty, especially for candidates who had calculated a constant of proportionality in part (i). Some very elegant solutions to part (ii) were seen, where the result emerged swiftly from $(165)^{2 / 3}$. The most absurd efforts came from candidates who made the implicit assumption that the Earth, Mercury and Neptune all travel at the same speed in their orbits, leading to wrong answers of 141 days and 165 respectively.
(a) Most students gave the answer to 3 significant figures, although 2 sf was what was required. The correct mass ( 3050 kg ) was chosen by those who used the correct formula, but some students used no mass in calculating the potential only.
(b) Many stated that the propellant/fuel was ejected through the nozzle. The statements about the momentum of the exhaust gases were often confused. The most popular way of deriving thrust was by attempting to use Newton's $3^{\text {rd }}$ law but the statements were often incomplete.
(c) The simple use of $\mathrm{F}=$ ma was easily achieved by most students.
(d) Although some students attempted to use conservation of momentum, most realised that the rocket equations was needed. There is the same confusion over the meaning of the symbols $v_{f}$ and $m_{f}$. Some used $m_{f}=1720 \mathrm{~kg}$ instead of 1330 kg , and others, after correctly calculating $v_{f}=996 \mathrm{~m} \mathrm{~s}^{-1}$, went on to subtract this from the exhaust gas speed, thus sacrificing a mark.
(e) (i) Most students chose the correct formula, but many forgot to square the radius, and others chose the wrong mass. The original mass of the spacecraft ( 3050 kg ) was the most popular erroneous value, although even the mass of the Earth was seen occasionally.
(ii) Nearly everyone started with the correct formula but two common errors ensued. Some forgot to take the square root and others did not convert 2.0 km to meters. Also some gave the answer to $1 \mathrm{sf}\left(0.9 \mathrm{~m} \mathrm{~s}^{-1}\right)$ thus losing a mark.
(iii) Those who calculated that the velocity change of the probe was $1.1 \mathrm{~m} \mathrm{~s}^{-1}$ followed with the right conclusion. Some students used the wrong mass but could still gain the third mark with a correct comparison.
(a) (i) Well done.
(ii) Candidates scored 2 or zero. The latter invariably used centripetal force $=$ gravitational force.
(iii) Many promising calculation were ruined by failure to cube the radius when finding the volume.
(b) Most candidates did not realise that escape velocity was not needed because the rocket was not escaping!

Most candidates scored the mark in part (a) (i) and went to use their answer correctly in part (ii). A small number of candidates however, failed to add the height calculated in part (i) to the Earth's radius or added the radius in km to the height in m . They were usually able to gain some credit for knowing the correct equation to use.

In part (b) (i), many candidates gave a clear and correct expression, using either the expressions for centripetal acceleration or the speed in terms of the mass of the Earth. Weaker candidates confused the symbols for speed and gravitational potential on the data sheet and attempted to calculate the speed using the expression for gravitational potential. Most candidates who completed part (i) went on to complete part (ii) successfully, although some lost the final mark as a result of giving the answer to too many significant figures. Some candidates in part (ii) successfully related the time period to the radius of orbit and thus gained full credit. A small minority of candidates gained no credit as a result of misreading part (b), attempting to provide answers based on a time period of 24 hours.

12 This question tested the gravitational inverse square law in the context of two planets orbiting a star. Application of Pythagoras' theorem shows that $(X Y)^{2}=3 R^{2}$. When the planets are closest, their separation is reduced to $R$; thus the force increases from $F$ to $3 F$. The facility of the question was $58 \%$, with one in five of the responses being for distractor $C(4 F)$.

13 This question turned to gravitational potential. At the mid-point $P$ between the two identical stars, the gravitational potential due to one of the stars must be $-G M / 0.5 d$, which is $-2 G M / d$. The total gravitational potential due to both stars must therefore be $-4 G M / d$. This was realised by $62 \%$ of the students. Faulty algebraic work probably caused $28 \%$ of the students to choose distractor B (-GM/d).

If calculated from first principles from the data given in the question, about the height of a geosynchronous satellite, it is a demanding question for the time available in an objective test. Nevertheless $53 \%$ of the responses were correct; perhaps some students had rehearsed the calculation and committed the result to memory. The question asked for the height of the satellite above the Earth's surface, and it is not surprising that the most common incorrect response was distractor C (the radius of the satellite's orbit). $24 \%$ of the students made this mistake.

